

# Exact final state integrals for strong field QED

Victor Dinu

Department of Physics, University of Bucharest, P. O. Box MG-11, Măgurele 077125, Romania

This paper introduces the exact, analytic integration of all final state variables for the process of nonlinear Compton scattering in an intense plane wave laser pulse, improving upon a previously slow and challenging numerical approach. Computationally simple and insightful formulae are derived for the total scattering probability and mean energy-momentum of the emitted radiation. The general form of the effective mass appears explicitly. We consider several limiting cases, and present a quantum correction to Larmor's formula.

*Introduction:* The recent and predicted progress in laser technology leading to very high peak intensities justify the need for a better understanding of so-called nonlinear QED, describing phenomena occurring in fields so strong that their effects cannot be treated perturbatively.

Unfortunately, the complexity of the processes inside these ultra-intense laser beams has meant that several simplifications have had to be used to make practical computations feasible. The laser beam is usually supposed to be in a coherent state, which can be well approximated by a classical field. Due to the relatively small frequencies, unless massive particles of very high Lorentz factor are involved, quantum effects are small, so a fully classical description may often be justified. For instance, in discussing the scattering of an electron in a laser beam, one may consider Thompson scattering instead of its quantum counterpart, nonlinear Compton scattering (NLCS) [1]. The classical approximation allows for a realistic description of the laser field and the inclusion of radiation reaction (RR) [2], but is unable to describe important quantum effects, such as nonperturbative pair creation from vacuum [3], the trident process [4], or vacuum birefringence [5].

A treatment of these processes in the framework of nonlinear QED, even in a semiclassical approach, has not yet been performed without further approximations, such as replacing the laser field by an idealized plane wave, thus allowing for analytical (Volkov) solutions to the Dirac equation. This disregards the strong spatial focusing of the beam needed to attain high intensities. In addition, for a long time, results were restricted to only infinite, monochromatic plane waves, or even, giving up periodicity, to a crossed field model [6, 7]. Only recently, the more realistic short pulse plane waves came into use [8–10].

In [8, 11, 12], the photonic and electronic distributions resulting from NLCS were described in detail for some model pulses. The total probability and emitted particle's energy-momentum can be obtained by integrating these distributions, but the task is numerically challenging. By a change in integration order and a different regularization, allowing for all final state integrals to be performed analytically, we obtain equivalent, but more revealing and much easier to compute formulae. For sim-

plicity, we only present unpolarized results. The method we develop is quite general and shall be applied to other strong field QED processes in a future paper.

*Preliminaries:* For our starting point, all notations and conventions, we refer the reader to [13]. In short,  $p = mv$  and  $k'$  are the initial electron and final photon four-momenta. We opt for natural units, so  $c = \hbar = 1$ . Define  $k = \omega n$ , where  $n^2 = 0$  and  $\omega$  is some characteristic frequency of the wave. Let  $\phi = k \cdot x = \omega x^-$  be an invariant *lightfront* coordinate, used to describe the plane wave pulse by the four-potential  $A = \frac{m}{e} a_0 f(\phi)$ . By transversality,  $k \cdot A = 0$ . We choose  $n = (1, \mathbf{e}_3)$ ,  $f_0 = f_3 = 0$ , and use lightfront notations such as  $p^\pm = p^0 \pm p^3$ ,  $\mathbf{p}^\perp = \mathbf{p} - p^3 \mathbf{n}$ . The final results for probability/momentum will prove to be manifestly Lorentz invariant/covariant. While working with  $f(\phi)$ , only the gauge changes keeping  $A$   $\phi$ -only dependent are allowed. But the end results can be expressed in terms of the classical velocity, so they are gauge invariant. For a long pulse, one may choose  $\omega$  the carrier frequency and the peak value of the envelope of  $f'(\phi)$  equal to one. To compare very short pulses one may prefer to fix  $\omega$  and  $a_0$  so that the pulse's  $\phi$  range is of order  $2\pi$  and  $-\int d\phi f'^2(\phi) = 1$ . Whatever our choice,  $a_0$  should offer a reliable description of the peak intensity, so  $\left| f'(\phi)^2 \right|_{\max} \sim 1$ .

*Classical motion:* Let  $\pi = mu$  be the kinetic momentum of a classical electron moving in this plane wave field, where

$$u(\phi) = v - a_0 f(\phi) + \frac{2a_0 f(\phi) \cdot v - a_0^2 f^2(\phi)}{2k \cdot v} k. \quad (1)$$

If we set  $f(-\infty) = 0$ , then  $v = u(-\infty)$  is indeed the velocity of the particle before meeting the wave. As opposed to unipolar pulses that permanently accelerate the particle [13], for the usual whole-cycle pulses,  $f(\infty) = 0$  and  $u(\infty) = u(-\infty)$ , as long as RR is neglected.

*Scattering probability:* In the Furry picture, the total NLCS probability for unpolarized initial electrons is:

$$P = \frac{-\alpha m^2}{4\pi^2 \omega^2 p^-} \int_0^{p^-} \frac{dk'^-}{k'^- (p^- - k'^-)} \int_{\mathbb{R}^2} d\mathbf{k}'^\perp \times \int_{\mathbb{R}^2} d\phi d\phi' \left[ 1 - a_0^2 g\left(\frac{k'^-}{p^-}\right) \theta^2 \langle f' \rangle^2 \right] e^{\frac{ik' \cdot (\pi) \theta}{\omega(p^- - k'^-)}}, \quad (2)$$

where  $\theta = \phi' - \phi$ ,  $g(\zeta) = \frac{1}{2} + \frac{\zeta^2}{4(1-\zeta)}$  and we denoted the moving average of a function  $F$  by  $\langle F \rangle = \theta^{-1} \int_{\phi}^{\phi'} F(\xi) d\xi$ . For instance,  $\langle f' \rangle = [f(\phi') - f(\phi)] / \theta$ . The formal expression (2) was obtained from formula C1 in [13], using the unregularized form of  $B_\nu = \int_{\mathbb{R}} \tilde{f}_\nu e^{-i\Phi} d\phi$ , with  $\tilde{f}_\nu \in (1, f_1, f_2, \mathbf{f}^2)$ , lightfront coordinates and a 4-dimensional integration. See also [8–10]. All  $B_\nu$  in the generic case, or at least  $B_0$ , need to be regulated, damping the oscillations of the integrand with a convergence factor such as  $e^{-\varepsilon\phi^2}$ ,  $\varepsilon \searrow 0$ , that can be discarded after a partial integration restricts  $B_\nu$  to the length of the pulse [8, 13].

At first glance, in writing (2) we have added to the numerical complexity, constructing the inner double integral from the simple integrals  $B_\nu$ . But, in fact, by a change of quadrature order, the analytical integration over  $\mathbf{k}'^\perp$ , and  $k'^-$  leaves us with only two easy integrals, instead of the initial four. In addition, we get rid of the rapid oscillations encountered when computing  $B_\nu$ . Expressing  $k' \cdot \langle \pi \rangle$  in lightfront coordinates, we notice the integral over  $\mathbf{k}'^\perp$  is Gaussian, if regulated by replacing in the exponent the factor  $\theta$  by  $\theta + i\varepsilon$ , then taking  $\varepsilon \searrow 0$ . The previous damping factor is now superfluous. Introducing the invariant number  $b_0 = \frac{2k \cdot v}{m}$ , the result is

$$P = \frac{-i\alpha}{\pi b_0} \int_{\mathbb{R}^2} d\phi d\phi' \int_0^1 d\zeta \frac{1 - a_0^2 g(\zeta) \langle f' \rangle^2 \theta^2}{\theta + i\varepsilon} e^{\frac{i\zeta\mu\theta}{(1-\zeta)b_0}}, \quad (3)$$

where

$$\mu = 1 + a_0^2 (\langle f' \rangle^2 - \langle f^2 \rangle) \geq 1. \quad (4)$$

We recognize the effective mass  $M = \sqrt{\langle \pi \rangle^2}$ , first introduced by Kibble, that appears in the Volkov propagator [14] and the Wigner function [15, 16]:

$$\mu = \langle u \rangle^2 = \frac{M^2}{m^2}. \quad (5)$$

A Lorentz and gauge invariant,  $M$  depends on the averaging interval. For a whole-cycle finite pulse, the mass shift  $M - m$  vanishes when  $\theta \rightarrow \infty$ . We now use the relation  $(\theta + i\varepsilon)^{-1} = p.v.\theta^{-1} - i\pi\delta(\theta)$  and the fact that  $\mu$  is an even function of  $\theta$ , so the result is indeed real. Changing variables from  $\phi, \phi'$  to  $\theta$  and  $\sigma = \frac{\phi + \phi'}{2}$ , and noticing that  $\pi = \int_{\mathbb{R}} dx \text{sinc } x$ , we get

$$P = -\frac{2\alpha}{\pi b_0} \int_{\mathbb{R}} d\sigma \int_0^\infty d\theta \int_0^1 d\zeta \times \left[ \frac{\partial \ln \mu}{\partial \theta} + a_0^2 \langle f' \rangle^2 \theta g(\zeta) \right] \sin \frac{\zeta\mu\theta}{(1-\zeta)b_0}. \quad (6)$$

A new analytical integration leads to:

$$P = -\frac{2\alpha}{\pi b_0} \int_{\mathbb{R}} d\sigma \int_0^\infty d\theta \times \left[ \frac{\partial \ln \mu}{\partial \theta} \mathcal{J}_1 \left( \frac{\mu\theta}{b_0} \right) + a_0^2 \langle f' \rangle^2 \theta \mathcal{J}_2 \left( \frac{\mu\theta}{b_0} \right) \right], \quad (7)$$

where, in terms of trigonometric integrals,

$$\begin{aligned} \mathcal{J}_1(x) &= -xA'(x), \quad \mathcal{J}_2(x) = \frac{1}{8} [2 + x - x^2 A(x)], \\ A(x) &= \sin x \text{ci}(x) - \cos x \text{si}(x), \\ A'(x) &= \cos x \text{ci}(x) + \sin x \text{si}(x). \end{aligned}$$

Notice that the Lorentz invariant (7) depends on  $b_0$ , and 4-products of the values of the function  $a_0 f$ , but not on  $a_0 f \cdot p$ . Interestingly, we can rewrite the probability in terms of the classical velocity (1) as function of the proper time, eliminating all reference to the driving field and proving gauge invariance. Could the result be generalized to an arbitrary, not necessarily plane, wave? It is hard to answer, because of the many very different trajectories allowed inside a general field. The attractiveness of a plane wave derives from the simplicity of the law of motion it entails. The electron's motion always looks the same, regardless of the initial position. A quantum computation for a general field would require the use of a wavepacket with some initial average position and momentum that only in the limit relates to a particular classical motion.

*Radiated energy-momentum:* The same procedure can be applied to compute the expectation value of the emitted photon's momentum,

$$\begin{aligned} \langle k'^\nu \rangle &= -\frac{\alpha m^2}{4\pi^2 \omega^2 p^-} \int_0^{p^-} \frac{dk'^-}{k'^- (p^- - k'^-)} \int_{\mathbb{R}^2} d\mathbf{k}'^\perp k'^\nu \\ &\times \int_{\mathbb{R}^2} d\phi d\phi' e^{\frac{ik' \cdot \langle \pi \rangle \theta}{\omega(p^- - k'^-)}} \left[ 1 - a_0^2 \langle f' \rangle^2 \theta^2 g \left( \frac{k'^-}{p^-} \right) \right]. \end{aligned} \quad (8)$$

Following the same method as for the probability, we are left with the manifestly covariant double integral

$$\langle k'^\nu \rangle = \frac{2\alpha}{\pi} \int_{\mathbb{R}} d\sigma \int_0^\infty d\theta (F \langle \pi^\nu \rangle + G k^\nu), \quad (9)$$

where

$$\begin{aligned} F &= \frac{\mathcal{J}_3 \left( \frac{\mu\theta}{b_0} \right) - \mathcal{J}_3 \left( \frac{\theta}{b_0} \right)}{b_0 \theta} - a_0^2 \langle f' \rangle^2 \frac{\theta}{b_0} \mathcal{J}_4 \left( \frac{\mu\theta}{b_0} \right), \\ G &= \frac{1}{b_0} \left[ \frac{2 - \mu}{\mu} \frac{\partial \mu}{\partial \theta} - 2 \frac{\mu - 1}{\theta} \right] \mathcal{J}_3 \left( \frac{\mu\theta}{b_0} \right) - a_0^2 \langle f' \rangle^2 \mathcal{J}_5 \left( \frac{\mu\theta}{b_0} \right). \end{aligned} \quad (10)$$

The new functions used are:

$$\begin{aligned} \mathcal{J}_3(x) &= \frac{x^2}{2} A(x) - x A'(x) - \frac{x}{2} \\ \mathcal{J}_4(x) &= \frac{1}{24} [(6x - x^3) A(x)]' + \frac{x}{12} \\ \mathcal{J}_5(x) &= \frac{1}{24} [(x^4 - 6x^2) A(x)]' - \frac{x^2}{8} + \frac{1}{3} \end{aligned}$$

The standard deviation of  $k'^\nu$  or any of its higher moments can similarly be computed. Even though derived

for a pulse, our formulae can provide the emission probability and radiated energy-momentum per cycle in the case of an infinite, periodic plane wave, by truncating it to only a finite number of cycles  $N$ , and considering the quickly reached  $N \rightarrow \infty$  limit of  $P/N$  and  $\langle k'' \rangle / N$ . The results are given by (7) and (9) with the  $\sigma$  integral restricted to one period. A simple example is a circularly polarized monochromatic wave  $\mathbf{f}(\phi) = (\cos \phi, \sin \phi, 0)$ , for which  $\mu = 1 + a_0^2 (1 - \text{sinc}^2 \frac{\theta}{2})$  is independent of  $\sigma$ , so  $\int d\sigma \rightarrow 2\pi$ .

Of particular interest are the limiting cases of small  $a_0$  or  $b_0$ . With today's technology,  $\omega$  is around the order of  $1 - 10^4$  eV and only for the optical range a large  $a_0$  is possible. If, for instance,  $\omega = 3$  eV, an electron energy of at least 20 GeV is needed for  $b_0$  to reach unity.

*Perturbative limit* The results can be expressed as integrals containing the field's Fourier transform and the Klein-Nishima probability of linear Compton [13]. To emphasize the role of  $b_0$ , we prefer to work with a quadratic function related to the potential's autocorrelation,

$$\mathcal{E}(\theta) = -a_0^2 \int_{\mathbb{R}} d\sigma \langle f' \rangle^2.$$

The weak field limits ( $a_0 \ll 1$ ) of (7) and (9) are:

$$P_p = \frac{2\alpha}{\pi} \int_0^\infty d\theta \mathcal{P}\left(\frac{\theta}{b_0}\right) \mathcal{E}(\theta), \quad (12)$$

$$\langle k'' \rangle_p = \frac{2\alpha}{\pi} \int_0^\infty d\theta \left[ p^\nu \mathcal{F}\left(\frac{\theta}{b_0}\right) + k^\nu \mathcal{G}\left(\frac{\theta}{b_0}\right) \right] \mathcal{E}(\theta), \quad (13)$$

where  $\mathcal{P}$ ,  $\mathcal{F}$ , and  $\mathcal{G}$  are the universal, positive functions,

$$\begin{aligned} \mathcal{P}(x) &= x^2 \int_x^\infty \frac{y-2x}{y^3} \mathcal{J}_1(y) dy + x \mathcal{J}_2(x), \\ \mathcal{F}(x) &= x^2 \int_x^\infty \frac{y-2x}{y^3} \mathcal{J}_3(y) dy + x \mathcal{J}_4(x), \\ \mathcal{G}(x) &= x^2 \int_x^\infty \frac{4x-3y}{y^3} \mathcal{J}_3(y) dy + \mathcal{J}_5(x). \end{aligned}$$

Since  $\mathcal{P}(x)$  is increasing for  $x > 0$ , 12 decreases with  $b_0$ , having the classical upper bound

$$P_{p, cl} = \frac{2\alpha}{3\pi} \int_0^\infty d\theta \mathcal{E}(\theta).$$

*Classical limit:* This is expected when  $b_0 \ll 1$ , since  $b_0$  is proportional to  $\hbar$ , a fact obscured by the use of natural units. That is, the laser photon energies are much smaller than  $m$  in the electron's rest frame. Now the major contribution to the integral (9) comes from small  $\theta$ . By Taylor expansions, such as

$$\mu = 1 + a_0^2 \left[ \frac{-f'(\sigma)^2}{12} \theta^2 + \mathcal{O}(\theta^4) \right], \quad (14)$$

one obtains the approximation

$$\langle k'' \rangle \simeq -\frac{\alpha}{3} b_0 a_0^2 \int_{\mathbb{R}} d\sigma \pi^\nu(\sigma) f'(\sigma)^2 \mathcal{C} \left[ -\frac{b_0^2 a_0^2 f'(\sigma)^2}{12} \right], \quad (15)$$

where

$$\mathcal{C}(x) = \frac{1}{\pi} \int_0^\infty [6\mathcal{J}_4(y + xy^3) - \mathcal{J}_3(y + xy^3)] y dy. \quad (16)$$

The above function is decreasing for  $x > 0$ , quickly departing from  $\mathcal{C}[0] = 1$ . Already  $\mathcal{C}[0.01] \simeq 0.58$ . No classical equivalent is found for the second term in the integrand of (9), negligible in this limit. An equivalent form of (15) can be written by expressing everything in terms of a classical particle's velocity as function of proper time,

$$\langle k'' \rangle \simeq -\frac{1}{6} \frac{e^2}{\pi} \int_{\mathbb{R}} d\tau u^\nu(\tau) \dot{u}(\tau)^2 \mathcal{C} \left[ -\frac{\dot{u}(\tau)^2}{3m^2} \right]. \quad (17)$$

When not only is  $b_0 \ll 1$ , but also  $b_0^2 a_0^2 \ll 1$ , equation (17) further reduces to:

$$\langle k'' \rangle_{cl} = -\frac{1}{6} \frac{e^2}{\pi} \int_{\mathbb{R}} d\tau u^\nu(\tau) \dot{u}(\tau)^2. \quad (18)$$

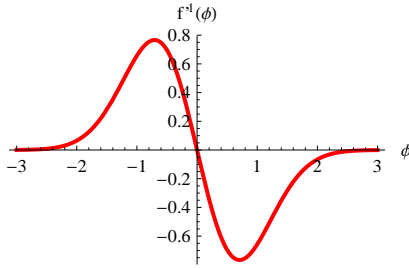
The same result arises directly from classical electrodynamics, if one neglects radiation reaction. For  $\nu = 0$ , (18) is just the time integrated Larmor's formula. Notice that a small  $b_0$  doesn't necessarily make classical Thompson scattering a good model. For strong fields, much less power is radiated than Larmor predicts. In CED, the spectral and angular distribution is also a double integral, but an interesting cancelling of interference terms leaves a single one after the frequency is integrated away. The transfer of energy and momentum to the field is well defined at each moment in time, there are no quantum uncertainties.

Interestingly, this form of decoherence is also shown by the better approximation (17), that looks misleadingly classical, though the argument of the correction  $\mathcal{C}$  is in fact proportional to  $\hbar^2$ . Heuristically,  $b_0$  is a sort of coherence lightfront time  $\theta$  scale. When small enough, it allows for a time-incoherent model of emission, but at high intensities the mass shift implied by (14) cannot be neglected in this coherence interval, hence the aforementioned correction. This happens when the peak electric field in the electron rest frame approaches the Schwinger critical value  $m^2/e$ . Formula (17) could provide a general improvement to Larmor's, valid in an arbitrary driving field, whose frequencies in the rest frame of the electron are similarly low compared to  $m$ , so the emission can be viewed as incoherent in time, hence the product of the classical motion of a charged particle. As for the total scattering probability, by an asymptotic expansion of the functions  $\mathcal{J}_i(x)$  one gets the limit

$$P_{cl} = \frac{2\alpha}{\pi} a_0^2 \int_0^\infty \frac{d\theta}{\theta^2} \int_{\mathbb{R}} d\sigma \frac{\langle f^2 \rangle - \langle f \rangle^2 - \frac{1}{2} \langle f' \rangle^2 \theta^2}{1 + a_0^2 (\langle f \rangle^2 - \langle f^2 \rangle)}. \quad (19)$$

In this case no decoherence is observed, as photon emission probability is not a classical concept.

*Numerical example:*



the shape of the electric field strength

We present results for a linearly polarized, one cycle, pulse, characterized by a Gaussian potential:

$$f^\mu(\phi) = \sqrt{\frac{2}{\pi}} \exp(-x^2) \delta_{\mu 1}, \quad (20)$$

The NLCS probability is shown in Fig.1. The quadratic increase from the perturbative region, shown in detail in the upper left corner, quickly slows down as  $a_0$  grows past unity. The higher the parameter  $b_0$ , the lower  $P$  is. In general, (7) boundlessly grows with the length/intensity of the pulse. Even for one as short as (20) and the experimentally attainable  $a_0 = 100$ , the result can easily surpass unity. In [17], this possibility was noticed, interpreted as a sign that multiphoton emission cannot then be neglected, and a re-normalization of the whole series of n-photon NLCS probabilities was suggested. Moreover, for a unipolar pulse, (7) shows the logarithmic IR divergence typical of Bremsstrahlung [13, 18]. These problems can be dealt with by including a one loop self-energy diagram, that adds nothing to (9), but does contribute to the expectation value of the final electron's momentum, even in the whole-cycle case [19]. For the general theory of the cancellation between real and virtual photon IR divergences, see [20].

To plot formula (9), we need more than just the invariants  $a_0$ ,  $b_0$ . Let's assume  $\omega = 1$  eV. It remains to know the incidence, that we choose head-on. While experimentally difficult, this gives the largest  $b_0$  for a given pulse and electron beam. In Fig.2 a comparison is drawn between the expectation value of the radiated energy and its two incoherent approximations. Both (18) and (17) overestimate (9), but the latter is a much closer match.

*Conclusions:* We have found a way to analytically integrate all final state variables out of the NLCS probability and expectation values. This not only saves huge computational expense, but also reveals new insights into the structure of scattering processes in strong fields. We shed light on the role of the effective mass and the emission's coherence in time. Simple results were found for the monochromatic, perturbative and classical limits. We derived a strong field correction to Larmor's formula, arising from the mass shift. A computation using a realistic model showed the usefulness of this approximation,

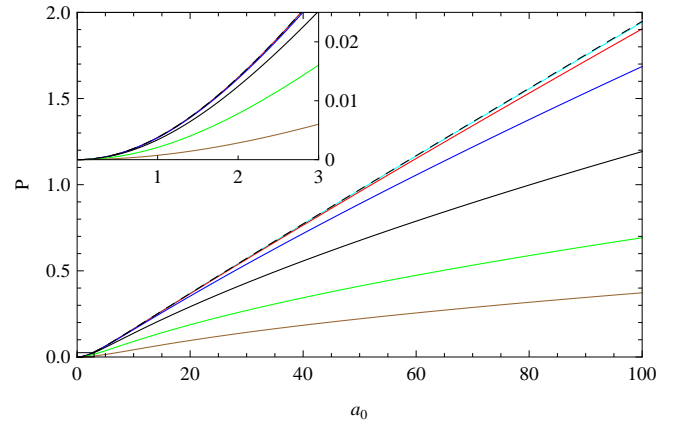


FIG. 1: The NLCS probability (7) is plotted as function of  $a_0$ , for  $b_0 = 10^p$ ,  $p = -4, 1$ . The dashed line is the classical limit (19).

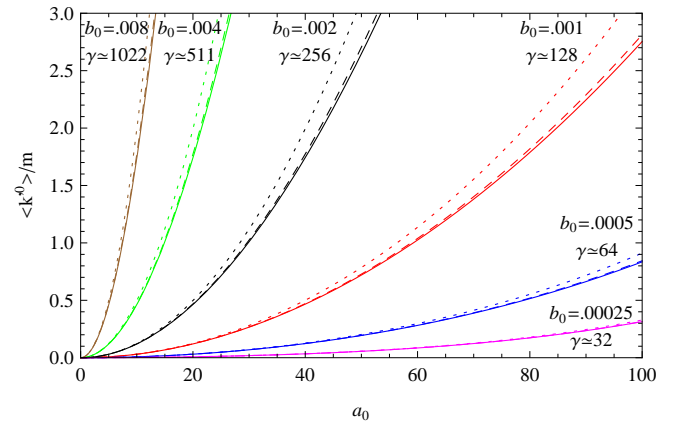


FIG. 2: The expectation value of  $\langle k^0 \rangle$ , as a multiple of the electron rest energy, plotted as function of  $a_0$ . The dotted line is the Thompson limit (18), while the dashed one is the better approximation (17).

but also found large values, even surpassing unity, for the probability. This suggests multiple scatterings and radiative corrections need to be considered. In a future paper, our method will be applied to these, as well as to other strong field processes. A thorough numerical exploration, comparison and interpretation of our results for various pulses and setups is in preparation.

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